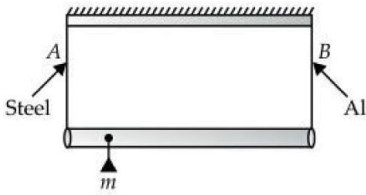


**TOPICS : Mechanics of Solids**

- Force constant of a spring ( $k$ ) is analogous to
  - $\frac{YA}{L}$
  - $\frac{YL}{A}$
  - $\frac{AL}{Y}$
  - $ALY$
- The Young's modulus of brass and steel are respectively  $1.0 \times 10^{11} \text{ N m}^{-2}$  and  $2.0 \times 10^{11} \text{ N m}^{-2}$ . A brass wire and a steel wire of the same length are extended by 1 mm each under the same force. If radii of brass and steel wires are  $R_B$  and  $R_S$  respectively, then
  - $R_S = \sqrt{2} R_B$
  - $R_S = \frac{R_B}{\sqrt{2}}$
  - $R_S = 4R_B$
  - $R_S = \frac{R_B}{2}$
- Modulus of rigidity of ideal liquids is
  - infinity
  - zero
  - unity
  - some finite small non-zero constant value
- Which of the statement is incorrect ?
  - Steel is more elastic than rubber.
  - The stretching of a coil is determined by its shear modulus
  - Stress is a vector quantity
  - Metal have larger values of Young's modulus than alloys and elastomers
- The ratio of tensile stress to the longitudinal strain is defined as
  - bulk modulus
  - Young's modulus
  - shear modulus
  - compressibility
- The average depth of Indian ocean is about 3000 m. The fractional compression,  $\frac{\Delta V}{V}$  of water at the bottom of the ocean is  
 (Given : Bulk modulus of the water =  $2.2 \times 10^9 \text{ N m}^{-2}$  and  $g = 10 \text{ ms}^{-2}$ )
  - 0.82%
  - 0.91%
  - 1.36%
  - 1.24%
- A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force stretches it along its length. The strain on the rod is  
 ( $Y_{\text{steel}} = 200 \times 10^9 \text{ N m}^{-2}$ )
  - 1.6 mm
  - 2.6 mm
  - 3.6 mm
  - 4.6 mm
- A wire of length  $L$  and radius  $r$  is clamped at one end. On stretching the other end of the wire with a force  $F$ , the increase in its length is  $l$ . If another wire of same material but of length  $2L$  and radius  $2r$  is stretched with a force  $2F$ , the increase in its length will be
  - $\frac{l}{4}$
  - $\frac{l}{2}$
  - $l$
  - $2l$
- Which of the following substances has highest value of Young's modulus ?
  - Aluminium
  - Iron
  - Copper
  - Steel
- Which one of the following statements are wrong ?
  - Young's modulus for a perfectly rigid body is zero
  - Bulk modulus is relevant for solids, liquids and gases
  - The young's modulus and shear modulus are relevant for solids
  - The stretching of a coil spring is determined by its shear modulus
- A rod of length  $L$  and negligible mass is suspended at its two ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in the figure. The cross-sectional areas of wires A and B are  $1 \text{ mm}^2$  and  $2 \text{ mm}^2$  respectively.  
 ( $Y_{\text{Al}} = 70 \times 10^9 \text{ N m}^{-2}$  and  $Y_{\text{steel}} = 200 \times 10^9 \text{ N m}^{-2}$ )
 

To have equal stress in both the wires, mass  $m$  should be suspended at a distance of

  - $\frac{1}{3}L$  from the wire A
  - $\frac{1}{2}L$  from the wire B
  - $\frac{2}{3}L$  from the wire B
  - $\frac{2}{3}L$  from the wire A

**TOPICS : Mechanics of Solids (SOLUTION)**

1. (a) :  $Y = \frac{FL}{A\Delta L}$  or  $F = \left(\frac{YA}{L}\right)\Delta L$   
Comparing this with  $F = k\Delta x$ , we get  $k = \frac{YA}{L}$

2. (b) : Increase in length  
 $\Delta L = \frac{FL}{YA} = \frac{FL}{Y\pi R^2}$   
As  $F, L$  and  $\Delta L$  are same hence,  
 $YR^2 = \text{a constant}$   
 $\therefore 2.0 \times 10^{11} R_S^2 = 1.0 \times 10^{11} R_B^2 \Rightarrow R_S = \frac{R_B}{\sqrt{2}}$

3. (b) : Modulus of rigidity of ideal liquids is zero.

4. (c) : Stress is not a vector quantity as it cannot be assigned a specific direction. It is a tensor quantity.

5. (b) : Young's modulus is defined as the ratio of tensile (or compressive) stress to the longitudinal strain.

6. (c) : The pressure exerted by a 3000 m column of water on the bottom layer is  
 $P = h\rho g$   
 $= 3000 \text{ m} \times 1000 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2}$   
 $= 3 \times 10^7 \text{ N m}^{-2}$

Fractional compression  $\frac{\Delta V}{V}$  is

$\frac{\Delta V}{V} = \frac{P}{B} = \frac{3 \times 10^7 \text{ N m}^{-2}}{2.2 \times 10^9 \text{ N m}^{-2}} = 1.36 \times 10^{-2} = 1.36\%$

7. (a) : Here,  $r = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$   
 $L = 1 \text{ m}$   
 $F = 100 \text{ kN} = 100 \times 10^3 \text{ N} = 10^5 \text{ N}$

Stress on the rod is given by

Stress =  $\frac{F}{A} = \frac{F}{\pi r^2}$   
 $= \frac{100 \times 10^3 \text{ N}}{3.14 \times (10^{-2} \text{ m})^2} = 3.18 \times 10^8 \text{ N m}^{-2}$

Elongation,

$\Delta L = \frac{(F/A)L}{Y} = \frac{(3.18 \times 10^8 \text{ N m}^{-2})(1 \text{ m})}{2 \times 10^{11} \text{ N m}^{-2}}$   
 $= 1.59 \times 10^{-3} \text{ m} = 1.59 \text{ mm}$

The strain on the rod is given by

Strain =  $\frac{\Delta L}{L}$   
 $= \frac{1.59 \times 10^{-3} \text{ m}}{1 \text{ m}} = 1.59 \times 10^{-3} = 1.59 \text{ mm}$   
 $= 1.6 \text{ mm}$

8. (c) : Let  $Y$  be the Young's modulus of the material of the wire. Then  
For the first wire

$Y = \frac{F/\pi r^2}{l/L} = \frac{FL}{\pi r^2 l}$  ... (i)

As both the wires are made of the same material, so their Young modulus is same.

Let the extension produced in second wire be  $l'$ . Then

$Y = \frac{2F/\pi(2r)^2}{l'/2L} = \frac{FL}{\pi r^2 l'}$  ... (ii)

Equating (i) and (ii), we get

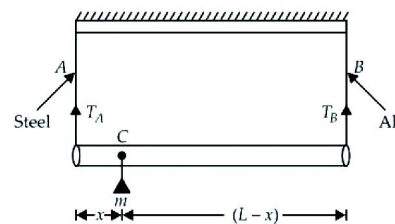
$\frac{FL}{\pi r^2 l} = \frac{FL}{\pi r^2 l'}$  or  $l' = l$

9. (d) :

Substance	Young's modulus $10^9 \text{ N m}^{-2}$
Aluminium	70
Copper	120
Iron	190
Steel	200

From the table, it is clear that among the given substances steel has the highest value of Young's modulus.

10. (d) : All other statements are correct.



Let mass  $m$  be suspended at a distance  $x$  from the wire  $A$ . Let  $T_A$  and  $T_B$  be tensions in the wire  $A$  (steel) and wire  $B$  (aluminium) respectively.

$\therefore$  Stress in wire  $A = \frac{T_A}{A_A}$

Stress in wire  $B = \frac{T_B}{A_B}$

For equal stress in both the wires,

$$\frac{T_A}{A_A} = \frac{T_B}{A_B}$$
$$\frac{T_A}{T_B} = \frac{A_A}{A_B} = \frac{1 \text{ mm}^2}{2 \text{ mm}^2} = \frac{1}{2} \quad \dots(i)$$

As the system is in equilibrium, taking moments about C, we get

$$T_A x = T_B(L - x)$$
$$\frac{L - x}{x} = \frac{T_A}{T_B}$$
$$\frac{L - x}{x} = \frac{1}{2} \quad \text{(Using (i))}$$
$$2L - 2x = x \quad \text{or} \quad x = \frac{2}{3}L$$